Probability Distribution of the Re-healing Delay in a One-Way Highway VANET

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Probability Distribution of the Re-healing Delay in a One-Way Highway VANET

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Abstract—**This letter proposes a closed-form expression for the probability distribution of the re-healing delay (time taken in the store-and-forward strategy to send a packet from a cluster head to the tail of the next cluster) conditioned on the gap distance between those two clusters on a one-way highway. Moreover, a closed-form expression is derived for the unconditional probability distribution of the re-healing delay. Using the derived probability distribution, one can straightforwardly study the impact of VANET parameters on the re-healing delay. Also, the probability distribution of the end-to-end delay in VANETs can be derived from the results in this letter. The accuracy of the proposed analysis is validated using simulations.**

I. INTRODUCTION

Vehicular ad-hoc networks (VANETs) enable ad-hoc communication between vehicles, or between vehicles and road-side units (RSUs) or drones [1]. VANET applications usually have time constraints on information dissemination. One example is emergency message (EM) dissemination (when an accident happens or a certain road condition is observed, an update should be broadcast as soon as possible).

VANETs can either be fully-connected, or sparsely-connected [2]. It is a crucial challenge to forward EM packets in sparsely-connected VANETs while satisfying the time constraints. A VANET is usually partitioned into a number of clusters (a cluster of vehicles is fully-connected and the distance between any two clusters is greater than the vehicle communication range). Fig. 1 shows an example of two clusters, where the head vehicle of the first cluster and the tail vehicle of the second cluster are labeled. Many proposed VANET routing protocols in the sparsely-connected case use a store-and-forward strategy (when the network is disconnected, packets can be stored and carried by a vehicle until reaching the next cluster). The re-healing delay is the time taken in the store-and-forward strategy to send a packet from a cluster head to the tail of the next cluster. This delay is the main focus of many vehicular active safety applications. Therefore, it is desirable to characterize the re-healing delay in VANETs. Moreover, the probability distribution of the re-healing delay can be used to derive the probability distribution of the end-to-end delay.

Several papers, such as [2]-[8], have analyzed the re-healing delay performance for safety message dissemination in VANETs. An expression for the mean of the re-healing delay was found in [2]. In addition, Ref. [3] derived the probability distribution of the re-healing delay based on an approximate probability distribution of cluster lengths. Morover, Ref. [4] proposed an accurate probability distribution of the cluster length, and from this derived the probability distribution of the re-healing delay. The authors in [2]-[4] assumed the same

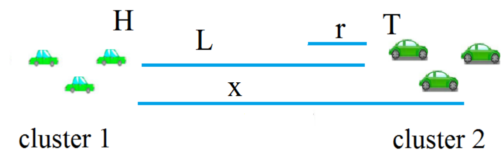

Fig. 1: Two clusters in a VANET.

system model (infrastructure-less bi-directional highway with a constant speed model for the vehicles).

Moreover, Refs. [5] and [6] calculated the mean of the re-healing delay to RSUs over a bi-directional highway. While, Ref. [5] considered a constant speed model, Ref. [6] considered a truncated normal distribution for vehicle speed.

On the other hand, Ref. [7] proposed an analytical model for the distribution of the re-healing delay. In addition, Ref. [8] derived closed-form expressions for the CDFs of the distances travelled by the cluster head and tail over time t, and these closed-form expressions were used to derive the probability distribution of the re-healing delay. Both Refs. [7] and [8] assumed the same system model (one-way highway with uniformly-distributed vehicle speeds).

One key difference between the system model in this work and that in many previous papers (e.g., [2] - [5]) is that we assume the speeds of the vehicles are uniformly distributed. Therefore, a vehicle can overtake another one travelling in the same direction. On the contrary, previous papers assumed a constant speed model for each direction, or the same speed for both directions. Therefore, clusters in the same direction could not re-heal except by using RSUs or by using vehicles moving in the opposite direction.

On the other hand, the key differences between this work, and [8] are as follows: 1) while this letter derives a closed-form expression for the CDF of the conditional re-healing delay, Ref. [8] does not provide a closed-form expression and its analytical method requires numerical integration, 2) in this letter we propose a correction for the derived expression proposed in [7] and [8] for the probability distribution of the conditional re-healing delay, and 3) in this letter we calculate closed-form expressions for the conditional and unconditional probability distributions of the re-healing delay.

The main contributions of this letter are as follows:

- It proposes a closed-form expression for the CDF of the re-healing delay conditioned on the given distance between two clusters in a VANET.
- It proposes a closed-form expression for the CDF of the unconditional re-healing delay in a VANET.
- It compares results from the proposed analysis with simulation results to show the accuracy of our analysis.

TABLE I: List of Notation

L	Gap between two consecutive clusters
r	Vehicle wireless communication range
x	R.V. for re-healing distance
$X(t)$	R.V. for distance cluster head moved in [0,t]
$X'(t)$	R.V. for distance cluster tail moved in [0,t]
T_c	R.V. for duration of re-healing phase
$F_{X(t)}(x)$	CDF of $X(t)$
$F_{X'(t)}(x)$	CDF of $X'(t)$
$f_{X(t)}(x)$	PDF of $X(t)$
$f_{X'(t)}(x)$	PDF of $X'(t)$
λ	Traffic flow rate (vehicles/unit time)
λ_s	Reciprocal of mean distance between vehicles
$v_{\min}$	Minimum allowed speed of vehicle
$v_{\max}$	Maximum allowed speed of vehicle
Δv	$v_{\max}$- $v_{\min}$
$u(\cdot)$	Heaviside unit step function
$\star$	Cross-correlation
s^*	Complex conjugate of s
$\mathscr{L}_x[\cdot]$	Laplace transform with respect to x
$\mathscr{L}_s^{-1}[\cdot]$	Inverse Laplace transform with respect to s

The rest of this letter is organized as follows. Section II introduces the system model. Section III gives a closed-form expression for the CDF of the re-healing delay conditioned on the given distance between two consecutive VANET clusters. Then, it proposes a closed-form expression for the unconditional probability distribution of the re-healing delay. Next, Section IV compares the simulation results against analytical results. Finally, the conclusions are presented in Section V.

II. System model

We consider a highway with vehicles moving in one direction. We assume that the speeds of the vehicles are uniformly distributed within the interval [$v_{\min}$, $v_{\max}$] [7], [8]. We also assume the inter-vehicular distances are exponentially-distributed, and that the inter-vehicular arrival rate is Poisson-distributed with a mean equal to the traffic flow rate λ (vehicles/sec) [7],[8].

In addition, the time required for a vehicle to receive and process a message before it is available for further relaying is neglected as in [4]-[8]. Also, Table I shows the list of notations used in our analysis. The medium access control (MAC) layer protocol is the distributed coordination function (DCF) of the IEEE 802.11. In addition, the radio channel propagation model is assumed to follow the Nakagami-m distribution [1]. Finally, the packet traffic model follows the constant bit rate (CBR) pattern.

III. Proposed model

In this section, we find a closed-form expression for the CDF of the re-healing delay conditioned on the distance between two clusters in one-way highway. Then, we derive a closed-form expression for the unconditional re-healing delay distribution.

Firstly, from [8], the closed forms for $F_{X(t)}(x)$ and $F_{X'(t)}(x)$ are equal to 0 for $x < v_{\min}t$ and 1 for $x > v_{\max}t$ and for x between $v_{\min}t$ and $v_{\max}t$, given by the following expressions

$$F_{X(t)}(x) = \frac{\frac{x}{t} - v_{\min}}{\Delta v} e^{\lambda\left(\frac{\frac{x v_{\min}}{v_{\max}} - v_{\min}t}{\Delta v} - t + \frac{x}{v_{\max}}\right)} \left(\frac{v_{\max}t}{x}\right)^{\frac{\lambda x}{\Delta v}},$$

and

$$F_{X'(t)}(x) = 1 - \frac{(v_{\max} - \frac{x}{t})\, e^{-\lambda\left(\frac{v_{\min}t - x}{v_{\max}}\right)} \left(\frac{x}{v_{\min}t}\right)^{-\lambda x/v_{\max}}}{\Delta v}.$$

Furthermore, from [7], the distribution of the re-healing delay T_c, conditioned on l, the distance between the cluster tail and the next cluster head, was derived. They obtained the following expression

$$P(T_c < t | L = l) = \int_0^{\infty} f_{X(t)}(x) \int_0^{x+r-l} f_{X'(t)}(x')\, dx'\, dx, \quad (1)$$

where $l > r$. However, both clusters are moving simultaneously. Therefore, the minimum period of time required for re-healing is $\left(\frac{l-r}{\Delta v}\right)$ seconds (when the cluster head moves with speed $v_{\max}$ and the next cluster tail with speed $v_{\min}$). Consequently, the distance the head must travel before re-healing is at least $(l-r)v_{\max}/\Delta v$. Hence, x in the previous expression should be corrected to start from $(l-r)v_{\max}/\Delta v$ not 0. Moreover, after considering this correction, the above equation can instead be expressed as

$$P(T_c < t | L = l) = \int_{(l-r)v_{\max}/\Delta v}^{\infty} f_{X(t)}(x)\, F_{X'(t)}(x + r - l)\, dx. \quad (2)$$

This can be identified as a windowed cross-correlation between $f_{X(t)}(x)$ and $F_{X'(t)}(x)$. From the theory of the Laplace transform, we know that the Laplace transform of the cross-correlation of two signals g and h is given by the expression $G^*(-s^*) \cdot H(s)$, where G and H are the Laplace transforms of g and h, respectively. Therefore, it is desirable to find the Laplace transforms of $f_{X(t)}(x)$ and $F_{X'(t)}(x)$.

The Laplace transform of $f_{X(t)}(x)$ is equal to $s \cdot \mathscr{L}_x[F_{X(t)}(x)](s) - F_{X(t)}(0)$. In addition, $F_{X(t)}(0) = 0$ because $F_{X(t)}(x)$ is zero when x less than $v_{\min}t$. Therefore, $\mathscr{L}_x[f_{X(t)}(x)](s)$ is equal to $s \cdot \mathscr{L}_x[F_{X(t)}(x)](s)$.

On the other hand, one can define the random variable X_1' such that $F_{X_1'(t)}(x) = 1 - F_{X'(t)}(x)$. Therefore,

$$\begin{aligned} P(T_c < t | L = l) &= \int_{\frac{(l-r)v_{\max}}{\Delta v}}^{\infty} f_{X(t)}(x)[1 - F_{X_1'(t)}(x + r - l)]\ dx \\ &= 1 - F_{X(t)}\left(\frac{(l-r)v_{\max}}{\Delta v}\right) - \int_{(l-r)v_{\max}/\Delta v}^{\infty} f_{X(t)}(x) F_{X_1'(t)}(x + r - l)\, dx \\ &= 1 - F_{X(t)}\left(\frac{(l-r)v_{\max}}{\Delta v}\right) - f_{X(t)} \star F_{X_1'(t)}(r - l), \\ &= 1 - F_{X(t)}\left(\frac{(l-r)v_{\max}}{\Delta v}\right) - \mathscr{L}_s^{-1}[s \cdot \mathscr{L}_x[F_{X(t)}(x)](s) \mathscr{L}_x[F_{X_1'(t)}(x)](s)](x), \end{aligned}$$

where

$$F_{X_1'(t)}(x) = \frac{(v_{\max} - \frac{x}{t})\, e^{-\lambda\left(\frac{v_{\min}t - x}{v_{\max}}\right)} \left(\frac{x}{v_{\min}t}\right)^{-\lambda x/v_{\max}}}{\Delta v}. \quad (3)$$

It can be found that

$$\mathscr{L}_x[F_{X(t)}(x)](s) = \frac{\dfrac{v_{\min} e^{-\left(\lambda t + \frac{\lambda v_{\min} t}{\Delta v}\right)}}{s - \frac{\lambda}{v_{\max}} + \frac{\lambda v_{\min}}{-v_{\max}^2 + v_{\min} v_{\max}}} - \dfrac{e^{-\left(\frac{\lambda v_{\max} t}{\Delta v}\right)}}{t\left(s - \frac{\lambda}{\Delta v}\right)^2}}{\Delta v}, \quad (4)$$

and

$$P(T_c<t|L=l)=\frac{v_{\min}-k_3/t}{\Delta v}e^{-\lambda\left(t+\frac{tv_{\min}v_{\max}-v_{\min}k_3}{v_{\max}\Delta v}-\frac{k_3}{v_{\max}}\right)}(tv_{\max}/k_3)^{\lambda k_3/\Delta v}\left(u(l-r)-u(k_{12})\right)+k_1k_4k_6\Big(\frac{k_9(v_{\max}^5v_{\min}-3v_{\max}^4v_{\min}^2+3v_{\max}^3v_{\min}^3-v_{\max}^2v_{\min}^4)}{\lambda^2k_8\Delta v}-\frac{k_{10}(v_{\max}^5v_{\min}-2v_{\max}^4v_{\min}^2+v_{\max}^3v_{\min}^3)}{\lambda^2k_8v_{\max}}+\frac{k_{10}k_{11}(v_{\max}^5-2v_{\max}^4v_{\min}+v_{\max}^3v_{\min}^2)}{\lambda k_2v_{\max}^2}+\frac{k_9k_{11}(-v_{\max}^5+3v_{\max}^4v_{\min}-3v_{\max}^3v_{\min}^2-v_{\max}^2v_{\min}^3)}{\lambda k_2(\Delta v)^2}\Big)/\left(t^2(\Delta v)^2\right)+\frac{v_{\max}v_{\min}k_1k_4k_5\left(v_{\max}k_9+k_{10}\Delta v\right)}{(\Delta v)^2(2v_{\max}-v_{\min})}+v_{\min}k_1k_4k_5\Big(\frac{v_{\max}^2k_9\Delta v}{\lambda k_2}-\frac{k_{10}k_{11}(-v_{\max}^3+v_{\max}^2v_{\min})}{v_{\max}^2(2v_{\max}-v_{\min})}+\frac{k_{10}(-v_{\max}^4+v_{\max}^3v_{\min})}{v_{\max}k_2\lambda}\Big)/k_7+v_{\max}k_1k_4k_6\Big(\frac{k_9(-v_{\max}^4+3v_{\max}^3v_{\min}-3v_{\max}^2v_{\min}^2+v_{\max}v_{\min}^3)}{\lambda k_2\Delta v}+\frac{k_{10}v_{\max}(\Delta v)^2}{\lambda k_2}-\frac{k_9k_{11}(v_{\max}^3-2v_{\max}^2v_{\min}+v_{\max}v_{\min}^2)}{(\Delta v)^2(2v_{\max}-v_{\min})}\Big)/k_7+k_{13},$$

where

$$k_1=u\left(-\frac{(l-r)(2v_{\max}-v_{\min})}{\Delta v}\right),\ k_2=(2v_{\max}-v_{\min})^2,\ k_3=l-r+v_{\min}t,\ k_4=e^{-\lambda v_{\min}t/v_{\max}},\ k_5=e^{(-\lambda t-\lambda v_{\min}t/\Delta v)},$$
$$k_6=e^{(-\lambda tv_{\max}/\Delta v)},\ k_7=t(\Delta v)^2,\ k_8=(2v_{\max}-v_{\min})^3,\ k_9=e^{k_{11}\lambda/\Delta v},\ k_{10}=e^{-k_{11}\lambda/v_{\max}},\ k_{11}=l-r+v_{\max}(l-r)/\Delta v,$$
$$k_{12}=l-r-t\Delta v,\ k_{13}=0.5+\text{piecewise}(v_{\min}\Delta vk_{12}\le 0,\ -0.5sign(k_{12}),\ 0<v_{\min}\Delta vk_{12},\ 0.5\,sign(v_{\max}t-v_{\max}(l-r)/\Delta v)). \quad (5)$$

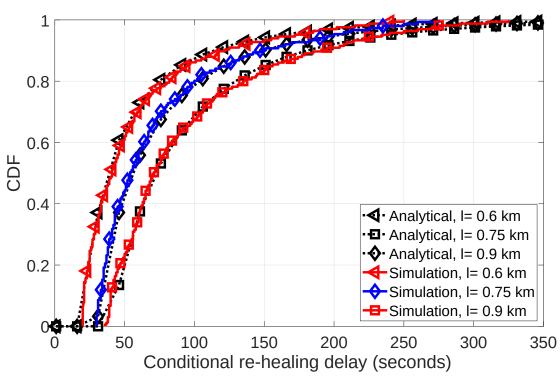

Fig. 2: Conditional re-healing delay with changing l.

$$\mathscr{L}_x[F_{X_1'(t)}(x)](s)=\frac{\frac{e^{-\left(\frac{\lambda v_{\min}t}{v_{\max}}\right)}}{t\left(s-\frac{\lambda}{v_{\max}}\right)^2}+\frac{v_{\max}e^{-\left(\frac{\lambda v_{\min}t}{v_{\max}}\right)}}{s-\frac{\lambda}{v_{\max}}}-}{\Delta v}. \quad (6)$$

Finally, the closed form for $P(T_c<t|L=l)$ is shown in Eq. (5).

A. CDF of the unconditional re-healing delay

From [7], the probability distribution of the unconditional re-healing delay was derived as follows

$$F_{T_c}(t)=P(T_c<t)=\int_0^\infty P(T_c<t|L=l)f_L(l)\,dl, \quad (7)$$

where $f_L(l)$ is the PDF of the inter-vehicular distances and is equal to $\lambda_s e^{-\lambda_s l}$, where λ_s is the reciprocal of the mean distance between vehicles and is equal to $2\lambda/(v_{\min}+v_{\max})$ as in [7] and [8].

This is of the same form as a Laplace transform if we assume $s=\lambda_s$. Therefore, F_{T_c} is the Laplace transform of $P(T_c<t|L=l)\lambda_s$, where λ_s is now the transform variable.

TABLE II: Simulation Parameters

Simulation Parameter	Values
λ (veh/s)	0.025, 0.035, 0.045
$v_{\min}$ (m/s)	15
$v_{\max}$ (m/s)	30
Simulation runs	500
Channel date rate (Mbps)	2
Communication range r (m)	300
Packet size (bytes)	512

Based on this form as a Laplace transform, the closed form for the CDF of the unconditional re-healing delay $P(T_c<t)$ can be calculated as shown in Eq. (8).

IV. Simulation and Model Validation

This section compares simulation results against analytical results using NS-2 (V-2.34). Also, we used VanetMobiSim [9] to generate realistic vehicle mobilities for a highway segment of length 30 km. We used greedy perimeter stateless routing protocol (GPSR) that greedily forwards the packets and added to it the store-and-forward strategy. In the simulations, one cluster is selected randomly to generate an emergency message, and the head of this cluster carries the message until it is connected to the next cluster. Table II summarizes the configuration parameters used in these simulations.

A. Conditional re-healing delay

Fig. 2 shows the analytical and simulation results for the CDF of the conditional re-healing delay with the same simulation parameters as in Table II and vehicular density λ equal to 0.025 veh/s, while changing the gap length l to values of (0.6, 0.75, and 0.9 km). It can be seen that the two curves (analytical, and simulation) agree closely across all re-healing delay values for the three vehicular densities, confirming the correctness and accuracy of the obtained closed-form expression.

$$P(T_c < t) = 2\lambda e^{-\left(k_2+k_1+\lambda t+\frac{2\lambda(r-v_{\min}t)}{v_{\max}+v_{\min}}\right)} \Bigg(v_{\min}e^{k_2} - v_{\max}e^{k_2} + v_{\max}e^{k_1+\lambda t} - v_{\min}e^{k_1+\lambda t} - \lambda t v_{\max}e^{k_1+\lambda t} + \lambda t v_{\min}e^{k_1+\lambda t} + \frac{2\lambda t e^{k_1+\lambda t}(v_{\max}^2+v_{\min}^2-2v_{\max}v_{\min})}{v_{\max}+v_{\min}} \Bigg) \Bigg/ \left(t(v_{\max}+v_{\min}) \left(\lambda + \frac{2(\lambda v_{\min}-\lambda v_{\max})}{v_{\max}+v_{\min}} \right)^2 \right) - e^{-\left(k_2-k_1+\frac{2\lambda r}{v_{\max}+v_{\min}}\right)} + 1 \tag{8}$$

where

$k_1 = 2\lambda v_{\min}t/(v_{\max}+v_{\min}),\ \ k_2 = 2\lambda v_{\max}t/(v_{\max}+v_{\min})$

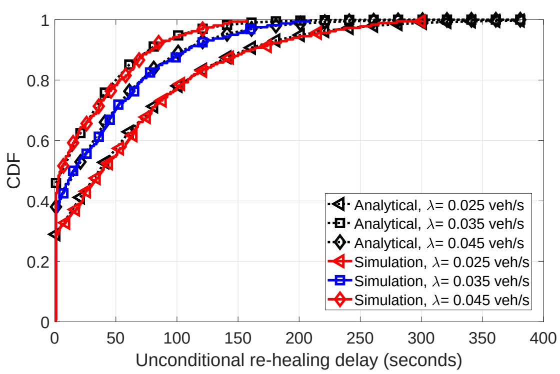

Fig. 3: Unconditional re-healing delay with changing λ.

Results show that the gap length highly impacts the CDF of the conditional re-healing delay. With increasing gap length, the CDF of the conditional re-healing delay decreases. This is because increasing the gap length causes an increase in the re-healing delay as we have the same speed in the three cases. Furthermore, it can be noted that the CDF of the conditional re-healing delay is equal to 0 for any values of t that are less than $(l-r)/\Delta v$ seconds. For instance, for a gap length equal to 0.6 km, the CDF of the conditional re-healing delay is equal to 0 for t less than 20 ((600-300)/15) seconds.

B. *Unconditional re-healing delay*

Fig. 3 shows the analytical and simulation results for the CDF of an unconditional re-healing delay with the same simulation parameters as in Table II, while changing the vehicular density λ to values of (0.025, 0.035, and 0.045) veh/s. Once again, the two curves (analytical, and simulation) agree closely across all re-healing delay values for the three vehicular densities.

Results show that the vehicular density significantly impacts the CDF of the unconditional single catch-up time. With increasing vehicular density, the CDF of the unconditional re-healing delay increases for all values of the vehicular density. This is because increasing the vehicular density causes an increase in the number of vehicles in each cluster. Consequently, there is a higher probability of a shorter overall re-healing delay. Moreover, increasing the vehicular density causes a decrease in the distance l between VANET clusters.

V. CONCLUSIONS

In this letter, we found a closed-form expression for the probability distribution of the re-healing delay conditioned on the gap distance between the clusters in a VANET. In addition, a closed-form expression for the unconditional probability distribution of the re-healing delay was derived. Extensive computer simulation results demonstrated the accuracy of the proposed analysis. The closed form for the unconditional re-healing delay characterizes the delay for broadcasting a safety message in a highway VANET. Also, it can be used to derive the end-to-end delay. From the closed form, we found out that the difference between $v_{\max}$ and $v_{\min}$ has a high impact on the re-healing delay. With increasing the speed difference Δv, the CDF of the conditional re-healing delay increases. This is because increasing Δv causes an increase in the relative speed between vehicles. Consequently, the re-healing delay for the same gap distance will be decreased. Due to space limitations, we did not include this plot in the letter. In addition, when the distance a from the current message head to a fixed RSU is less than twice the communication range r, i.e., $a < 2r$, and there is no direct communication path to the RSU at the initial time, then the conditional re-healing time to the next cluster can be used to find the CDF of the total end-to-end delay in this special case. Due to space limitations, we leave this to future work, although this provides additional evidence of the applicability of the current work to practical scenarios. In our future work, we will consider calculation of the probability distribution of the re-healing delay for a two-way highway.

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