Accurate Probability Distribution Calculation for Drone-Based Highway-VANETs

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4 authors, including:

Hafez Seliem
Memorial University of Newfoundland

Reza Shahidi
Memorial University of Newfoundland

Mohamed Hossam Ahmed
University of Ottawa

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Hafez Seliem, Reza Shahidi, Mohamed H. Ahmed, and Mohamed S. Shehata
Email: {hms117, d97rs, mhahmed, mshehata}@mun.ca

***Abstract*—In this letter, we analytically derive the probability distribution of the vehicle-to-drone packet delay on a bi-directional highway. The model on which the analysis is based considers the wireless communication range of the vehicles and the cluster length. In addition, the proposed analysis finds that the same calculation in related work underestimates the maximum inter-drone distance, stochastically limiting the vehicle-to-drone packet delay using the drone active service (DAS). Simulations are used to validate the proposed analysis.**

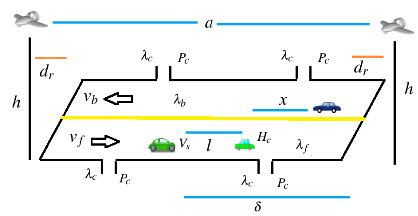

Fig. 1: System model for the considered highway.

I. Introduction

A vehicular ad-hoc network (VANET) is a mobile ad-hoc network among vehicles, or vehicles and other infrastructure units (road-side units or drones). VANETs have many applications as safety applications used to avoid collisions. Moreover, VANETs have commercial and comfort applications. In the case of safety applications, there are constraints on the packet delivery delay where a message should be received within a certain time threshold. For this reason, it is important to analyze the statistical characteristics of the packet delivery delay for safety messages in VANETs such as the probability distribution function and moments.

There are many papers that have analyzed the probability characteristics of the vehicle-to-infrastructure (V2I) packet delay in VANETs. For instance, the expectation of the packet delivery delay between Internet access points and a vehicle was derived in [1]. Moreover, the expected value of the vehicle-to-RSU delay was derived analytically in [2]. Furthermore, the authors in [3] proposed a closed-form expression for the expected delay of broadcast alert messages up to reception by the nearest RSU in a highway VANET.

In addition, the authors in [4] presented a probabilistic analysis using effective bandwidth theory for the vehicle-to-RSU packet delivery delay where RSUs were uniformly distributed over the highway. Furthermore, a closed-form expression for the cumulative density function (CDF) of the vehicle-to-RSU packet delivery delay in the worst case was proposed in [5]. Moreover, based on drones that are uniformly distributed over the highway, a closed-form expression for the CDF of the vehicle-to-drone packet delay in the worst-case was proposed in [6]. Also, they accounted for the drone wireless communication range and proposed a drone active service (DAS) that updates the inter-drone distance.

The main difference between this proposed analysis and those in the previously-mentioned works (e.g., [1] - [5]) is that we focus on the vehicle-to-drone delay's probability distribution, not just the expected value. Using the proposed analysis, we can calculate the maximum inter-drone distance which stochastically limits the delay to a certain upper bound. Consequently, the minimum number of drones required to be uniformly distributed over a two-way highway to satisfy such probabilistic constraints can be determined.

On the other hand, the main difference between this work and that in [5] and [6] is that our analysis considers the vehicles' communication range and the VANET cluster length. Therefore, our analysis is more accurate, especially for higher values of the vehicular density where the vehicle can forward the packet to the next neighboring vehicle. On the contrary, [5] and [6] focus on the worst case only when the message is carried by its original source vehicle until arriving within the wireless communication range of the nearest infrastructure unit (RSUs in [5] or drones in [6]). Therefore, they did not consider the scenario where the packet can be forwarded between the packet's source vehicle and the next infrastructure drone.

The main contributions of this letter are as follows: 1) it proposes an analytical expression for the vehicle-to-drone packet delay probability distribution, and (2) it presents the comparison between results from our analysis with simulation results and previous work to validate our analysis.

The rest of this letter is organized as follows. Section II presents the system model. Section III presents the proposed analysis and the obtained expression. Then, Section IV compares the proposed analysis results against simulation results and previous work. Finally, in Section V, conclusions and future work are given.

II. System model

We use the same system model proposed in [5] and [6]. However, in this model, we take into consideration the vehicle wireless communication range and vehicle's cluster length to derive a more accurate probability distribution of the vehicle-to-drone packet delivery delay. In our model, we consider a bi-directional highway as shown in Fig. 1. Moreover, for each segment of length a, we have two drones with a wireless communication d_r, one at each end, as in [6]. In addition, we

TABLE I: List of Notation

a	Inter-drone distance
d_r	Communication range for the drone
X	R.V. representing the inter-vehicle distance
h	Altitude for the drone
v_f	Forward direction speed
v_b	Backward direction speed
Δ	R.V. representing the distance between the drone and a vehicle
r	Vehicle communication range
P_c	Probability the vehicle leaves the highway at any junction
λ_c	Expected number of junctions
$u(\cdot)$	Heaviside unit step function
λ_f	Forward direction exponential rate parameter
λ_b	Backward direction exponential rate parameter

assume the vehicles in each direction are moving with constant speeds of v_f and v_b in the forward and backward directions, respectively (the forward direction is the direction towards the final destination, and the backward direction is the opposite direction). Furthermore, we assume the VANET consists of a group of one or more disconnected VANET clusters. A VANET cluster consists either of a single vehicle not within the communication range of any other vehicle, or a maximal set of vehicles in which every pair of vehicles in the cluster is connected by at least one multihop path [7]. Furthermore, the Poisson distribution is assumed for the number of vehicles in each direction and the exponential distribution is assumed for the inter-vehicular distances [6]. Moreover, y is the distance in the forward direction between the packets source vehicle and the next infrastructure drone.

Moreover, with a probability P_c, we assume a vehicle can exit at any road junction. Furthermore, we assume the number of road junctions is Poisson-distributed with parameter λ_c as shown in Fig. 1. Furthermore, we assume that any vehicle can be the source of packets and an infrastructure drone is the destination. In addition, one replica of the message is sent in the opposite direction of the highway (the direction opposite to that for the source vehicle direction) by the cluster head in the forward direction. Consequently, packets are forwarded to the drone either by the cluster head in the forward direction or a moving vehicle in the opposite direction.

III. Proposed analysis

The proposed analysis follows the same methodology of the analysis proposed in [6]. However, in this letter, the vehicle wireless communication range r is considered, which was not the case in the model and analysis in [6]. In this letter, our goal is to derive an analytical expression for the CDF of the vehicle-to-drone packet delay in terms of r (vehicle wireless communication range), a (inter-drone distance), d_r (drone wireless communication range), λ_c (reciprocal of mean distance between those junctions), and λ_f and λ_b (forward and backward directions exponential rate parameters), as depicted in Fig. 1. Then, using the proposed analysis, one can obtain the minimum number of infrastructure drones corresponding to the maximum value of the inter-drone distance (a) that stochastically limits the vehicle-to-drone packet delay to a certain upper bound with a violation probability of at most ε, which can be expressed as follows

$$\begin{aligned} &\text{maximize} \quad a \\ &\text{subject to} \quad 1 - F_T(T_{\max}, a) \leq \varepsilon. \end{aligned} \tag{1}$$

In addition, we consider that the cluster head forwards the packet in the opposite and forward directions and the analytical calculation considers the packet received earlier by the destination (infrastructure drone). In this case, the vehicle-to-drone packet delay will be lower than that if the cluster head just sent the packet in the forward direction. Furthermore, we can represent the probability that the vehicle-to-drone packet delay T is lower than a value t as follows

$$Pr(T \leq t) = Pr(T \leq t, C = 0) + Pr(T \leq t, C = 1), \tag{2}$$

where C is a random variable for the number of replicas forwarded from the packet. Therefore, the term $Pr(T \leq t, C = 0)$ is the joint cumulative probability the cluster head forwards the original message in the forward direction only, in a given time interval [0,t]. On the contrary, the term $Pr(T \leq t, C = 1)$ is the joint cumulative probability the cluster head forwards one extra replica of the message in the opposite direction besides sending the original message in the forward direction, in a given time interval [0,t].

We first derive an analytical expression for the first term $Pr(T \leq t, C = 1)$. The joint probability of the vehicle-to-drone packet delay T and the number of replicas C can be represented as follows

$$Pr(T \leq t, C) = Pr(T \leq t|C) Pr(C). \tag{3}$$

On the other hand, as shown in Fig. 1, δ, the distance in the forward direction between the source vehicle V_s and the next drone, is between 0 and $a - 2d_r$, where d_r is the drone wireless communication range. On the contrary, X (the distance between the cluster head in the forward direction and the first moving vehicle that receives the packet from the cluster head in the opposite direction) lies between 0 and ∞. In addition, l, the cluster length (the distance between the the source vehicle V_s and the cluster's head (H_c) is between 0 and ∞. Consequently, we can represent Eq. (3) for the case C=1 as follows

$$\begin{aligned} Pr(T \leq t, C = 1) = \int_0^\infty f(l) \int_0^\infty \int_0^{a-2d_r} & Pr(T \leq t | C = 1, X = x, \Delta = \delta) \\ & Pr(C = 1 | X = x, \Delta = \delta) f(\delta, x) \; d\delta \; dx \; dl, \end{aligned} \tag{4}$$

Furthermore, the term $Pr(T \leq t | C = 1, X = x, \Delta = \delta)$ is the conditional probability of the vehicle-to-drone packet delay in the case when the packet is transmitted in the forward and backward directions. This term can be expressed as follows

$$\begin{aligned} & Pr(T \leq t | C = 1, \Delta = \delta, X = x) = \\ & \quad u\left(t - \min\left(\frac{a - \delta - d_r + x}{v_b}, \frac{\delta - d_r - l}{v_f}\right)\right). \end{aligned} \tag{5}$$

Moreover, $\min(\cdot)$ in Eq. (5) can be removed as follows

$$= \begin{cases} u\left(t - \dfrac{\delta - d_r - l}{v_f}\right) & \text{if } 0 \leq \delta \leq \min(b_1, a - 2d_r) \\ u\left(t - \dfrac{a - \delta - d_r + x}{v_b}\right) & \text{if } \min\left(b_1, a - 2d_r\right) \leq \delta \leq a - 2d_r, \end{cases}$$

$$\text{where} \quad b_1 = \frac{(a - d_r + x)v_f + (l + d_r)v_b}{v_b + v_f}. \tag{6}$$

TABLE II: Parameters of the simulation

Parameter	Value
a (km)	5, 6.5, 8
P_c	0.02
v_b (m/s)	30
v_f (m/s)	25
λ_c	0.002
Simulation runs	600
r (m)	300
d_r (m)	550
Simulation time (seconds)	800

Therefore,
$Pr(T \le t|C=1) =$

$$\int_0^{\infty} f(l) \int_0^{\infty} \int_0^{min(b_1,a-2d_r)} f(\delta,x) u\left(t - \frac{\delta - d_r - l}{v_f}\right) f(\lambda_c,P_c)\, d\delta\, dx\, dl$$
$$+ \int_0^{\infty} f(l) \int_0^{\infty} \int_{min(b_1,a-2d_r)}^{a-2d_r} f(\delta,x) u\left(t - \frac{a - \delta - d_r + x}{v_b}\right) f(\lambda_c,P_c)\, d\delta\, dx\, dl. \quad (7)$$

Moreover, $f(\delta,x)$ term is representing the joint probability density function (PDF) of X and Δ. In addition, as mentioned previously in the system model, the source vehicle location is uniformly-distributed over the distance a, i.e., the random variable Δ is uniformly-distributed. On the other hand, as the vehicles form a Poisson process, X is exponentially-distributed. Consequently, $f(\delta,x)$ can be formulated as follows

$$f(\delta,x) = \frac{\lambda_f + \lambda_b}{a - 2d_r} e^{-(\lambda_f+\lambda_b)x}, \quad 0 \le \delta \le a - 2d_r, x > 0. \quad (8)$$

In addition, $f(\lambda_c,P_c)$ is the probability that the cluster head vehicle does not exit at any road junction over the highway before arriving within the wireless communication range of the next infrastructure drone. The authors in [6] obtained the expression for this probability as follows

$$f(\lambda_c,P_c) = e^{-\lambda_c P_c (a-\delta-2d_r+x)}. \quad (9)$$

In addition, the PDF of the cluster length is derived in [7] as follows

$$f(l) = \frac{\lambda_f}{e^{-\lambda_f r} - 1} \sum_{i=0}^{\lfloor l/r \rfloor} \frac{\left(-\lambda_f (l - i\,r)\right)^{i-1}}{-m!} \left(\lambda_f (l - i\,r) + m\right) e^{-\lambda_f i\,r}. \quad (10)$$

On the other hand, in the case of $C=0$, by following the same methodology for the analysis of the $C=1$ case, the expression for the CDF in that case can be formulated as follows
$Pr(T \le t|C=0) =$

$$\int_0^{\infty} f(l) \int_0^{\infty} \int_0^{a-2d_r} f(\delta,x) u\left(t - \frac{\delta - d_r - l}{v_f}\right) f(\lambda_c,P_c)\, d\delta\, dx\, dl, \quad (11)$$

where $f(\lambda_c,P_c)$ is $1 - e^{-\lambda_c P_c (a-\delta-2d_r+x)}$ as in [5] and [6].

Using Eqs. (2), (3), (7), and (11), the CDF of vehicle-to-drone packet delay can be simplified as represented in Eq. (12).

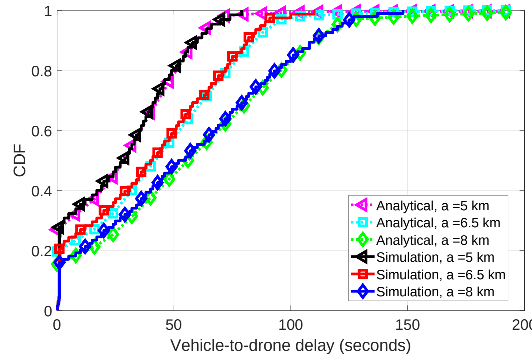

Fig. 2: Inter-drone distance a and the CDF of the delay.

IV. Simulation and Model Validation

The proposed system model is implemented in NS-2 (v. 2.34). Moreover, VanetMobiSim [8] is used to generate vehicle mobility scenarios. A summary for the simulation parameters is presented in Table II.

A. Inter-drone distance

With the parameter values in Table II, the simulation and analytical results for the proposed analysis are shown in Fig. 2, while varying the inter-drone distance a to values of (5, 6.5, and 8) km.

One can note that that the two curves (simulation, and analytical) agree closely for the three inter-drone distances across all delay values, reflecting the correctness and accuracy of the proposed model. In addition, the results show the impact of the inter-drone distance parameter a on the CDF of the vehicle-to-drone delay. When the inter-drone distance a decreases, the CDF values increases. This is because increasing a causes the vehicle to carry the packet for a longer distance and we have the same speed in the three cases. Therefore, the vehicle-to-drone packet delay increases.

B. Proposed analysis and previous work comparison

As mentioned in Section I, a closed-form expression for the vehicle-to-drone packet delay probability distribution in the worst case was proposed in [6]. On the other hand, the analysis here yields a more accurate probability distribution by considering the VANET cluster length and wireless communication range for vehicles. With the parameter values in Table II, the analytical results of the proposed analysis and those from [6] are shown in Fig. 3, while changing the vehicular densities λ_f and λ_b to values of (0.002, 0.005, 0.008) vehicle/m.

It can be noted that in the results for the three values of the vehicular densities, the CDF values of our model are higher than those of [6]. This is expected, as [6] took into consideration the worst case only. In addition, the results show that there is a big difference between both analyses especially at higher vehicular density values. At lower vehicular density values (like 0.002 veh/m), the difference between the two curves is smaller. This is because at lower vehicular density values, the cluster length is shorter and the cluster head carries the packets for a longer distance (very close to the worst case where the cluster head is the vehicle source as in [6]). On the

$$Pr(T \le t) = \sum_{k=0}^{\left\lfloor \frac{b_3}{r} \right\rfloor} \int_0^r f(kr+l) \int_0^{b_2} \int_0^{b_1} f(\delta,x) u\left(t - \frac{\delta - d_r - l}{v_f}\right) f(\lambda_c, P_c)\ d\delta\ dx\ dl + \sum_{k=0}^{\left\lfloor \frac{b_3}{r} \right\rfloor} \int_0^r f(kr+l) \int_0^{b_2} \int_{b_1}^{a-2d_r} f(\delta,x)$$

$$u\left(t - \frac{a-\delta-d_r+x}{v_b}\right) f(\lambda_c,P_c)\ d\delta\ dx\ dl + \sum_{k=0}^{\infty} \int_0^r f(kr+l) \int_0^{\infty} \int_0^{a-2d_r} f(\delta,x) u\left(t - \frac{\delta - d_r - l}{v_f}\right)\left(1 - e^{-\lambda_c P_c (a-\delta-2d_r+x)}\right)\ d\delta\ dx\ dl,$$

where

$$b_1 = \frac{(a-d_r+x)v_f + (kr+l+d_r)v_b}{v_b+v_f},\ b_2 = \frac{(a-3d_r-kr-l)v_b - v_f d_r}{v_f},\ b_3 = \frac{(a-3d_r)v_b - v_f d_r}{v_b} \tag{12}$$

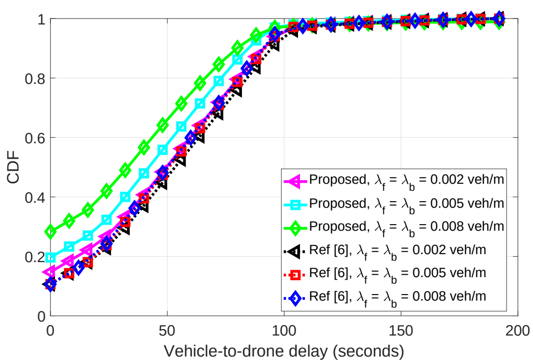

Fig. 3: Results from proposed analysis vs. those from Ref. [6].

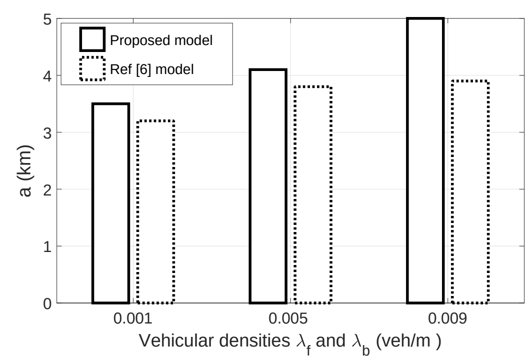

Fig. 4: DAS Results compared with those from Ref. [6].

other hand, at higher vehicular densities, the differences are very high. This is because the probability of having a longer cluster length increases. Consequently, the cluster head carries the packet for a shorter time. Moreover, the results show that the vehicular densities have a lower impact on the CDF of the delay in [6]. This is because in [6], the vehicular density has an impact only in the opposite direction on the random variable x.

C. Drone-active service results

Fig. 3 shows the DAS simulation results of the proposed analysis and those from [6] with the parameter values in Table II, and based on Eq. 1 where ε equal 0.05 and $T_{\max}$= 50 seconds, while changing the vehicular densities λ_f and λ_b to values of (0.001, 0.005, 0.009) veh/m.

Results show that the DAS calculation in our case always yields a higher inter-drone distance a than that from the DAS calculation from [6] for the same parameters. Consequently, our analysis requires a lower number of drones to cover the highway than that required in the case of the analysis from [6]. At lower values of vehicular densities, this difference decreases. This is because at lower vehicular density values, the cluster lengths are shorter. On the contrary, at higher vehicular density, the differences are very high. This is because the probability of having a longer cluster length increases.

V. CONCLUSION

In this letter, we proposed an analytical expression for the vehicle-to-drone packet delay probability distribution on a bi-directional highway. This analysis is more accurate than in previous works that focus on the worst case only. The drone-active service (DAS) can benefit from our analysis. Our analysis is more accurate and the CDF from this analysis is always higher than that proposed in [6]. Also, the DAS calculation included here requires a lower number of drones than that required in the case of the analysis from [6]. In future work, infrastructure-less drones can be considered.

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